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$\mu\tau$ symmetry, tribimaximal mixing and four zero neutrino Yukawa textures

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Abstract: Within the type-I seesaw framework with three heavy right chiral neutrinos and in the basis where the latter and the charged leptons are mass diagonal, a near $\mu\tau$ symmetry in the neutrino sector is strongly suggested by the neutrino oscillation data. There is further evidence for a close to the tribimaximal mixing pattern which subsumes $\mu\tau$ symmetry. On the other hand, the assumption of a (maximally allowed) four zero texture in the Yukawa coupling matrix Y_ν in the same basis leads to a highly constrained and predictive theoretical scheme. We show that the requirement of an exact $\mu\tau$ symmetry, coupled with observational constraints, reduces the *seventy two* allowed textures in such a Y_ν to *only four* corresponding to just two different forms of the light neutrino mass matrix m_ν. The effect of each of these on measurable quantities can be described, apart from an overall factor of the neutrino mass scale, in terms of two real parameters and a phase angle all of which are within very constrained ranges. The additional input of a tribimaximal mixing reduces these three parameters to **only one** with a very nearly fixed value. Implications for both flavored and unflavored leptogenesis as well as radiative lepton flavor violating decays are discussed. We also investigate the stability of these conclusions under small deviations due to renormalization group running from a high scale where the four zero texture as well as $\mu\tau$ symmetry or the tribimaximal mixing pattern are imposed.

Keywords: Neutrino Physics, Beyond Standard Model

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Contents

1 Introduction

A lot is now known [1] about the masses and mixing angles of the three light neutrinos, based on the solid foundation of accumulated experimental evidence, while the remaining gaps are expected to be filled in the forseeable future. Thus the task of pinning down the form of their Yukawa coupling matrix Y_ν in flavor space, assuming the existence of three heavy right chiral neutrinos, is very much at hand. The general structure of Y_ν is, however, intractable at the moment. One needs concrete theoretical ideas to simplify it and then test such simplified forms by comparing with extant data. Our present work is in such a spirit.

We try in this paper to bring together three theoretical ideas:

(1) allowed four zero neutrino Yukawa textures [2]–[3],

(2) $\mu\tau$ symmetry [4]–[30] and

(3) a tribimaximal mixing pattern[1] [31]–[34], which actually subsumes the results of (2).

Within the type-I seesaw framework [37]–[40] and in the weak basis where the charged leptons l_α (α = 1,2,3) and the heavy right chiral neutrinos N_i (i=1,2,3) have real and diagonal respective masses m_α and M_i, we explore the mutual compatibility between (1) and (2) and further between (1) and (3). A drastic reduction of the allowed textures and parameters under (1) ensues.

Let us start with (1). Assuming the absence[2] of any strictly massless neutrino as well as that of any unnatural cancellation, the utilization of the observed lack of complete decoupling of any neutrino flavor from the two others led to the demonstration [2] that four is the maximum number of zeroes allowed in Y_ν. All allowed four zero textures, seventy two

[1] Such a pattern could be due to a flavor symmetry in the Lagrangian such as A_4 [35], S_3 [36].

[2] Allowing one massless neutrino, five zeroes are allowed in Y_ν [41]–[42].

configurations in total, were completely classified in [2] into two categories: (A) fifty four textures with two (element by element) orthogonal rows i and j say; (B) eighteen textures with nonorthogonal rows and one row having two zeroes with the other two rows (k and l, say) having one zero each. Let us write the complex symmetric light neutrino Majorana mass matrix in our basis as

$$m_\nu = -Y_\nu \text{ diag.}(M_1^{-1}, M_2^{-1}, M_3^{-1})\ Y_\nu^T v^2, \tag{1.1}$$

v being the relevant Higgs VEV. Now, for all textures of category (A), one has the condition [2]

$$(m_\nu)_{ij} = 0 : \text{category (A)}, \tag{1.2}$$

while, for those of category (B), the condition is [2]

$$\det \text{ cofactor}[(m_\nu)_{kl}] = 0 : \text{category (B)}. \tag{1.3}$$

One very important and interesting feature of all these allowed four zero textures is that they enable [2] the complete reconstruction of the neutrino Dirac mass matrix $m_D = vY_\nu$ in terms of the physical masses of the light neutrinos as well as $M_{1,2,3}$ and the elements of the unitary PMNS mixing matrix including the Majorana phase matrix factor. This means [2] that the high scale CP violation required for leptogenesis gets specified exclusively [43]–[48] in terms of the CP violation pertaining to laboratory energy neutrinos. Another striking feature of these textures is the following. Conditions (1.2) and (1.3) on the corresponding neutrino mass matrix m_ν are invariant [3] under renormalization group running at the one loop level, though texture zeroes in general are not. Thus if these conditions are the consequences of some symmetry operative at a high scale, they would be approximately valid *even at laboratory energies* where neutrino oscillation experiments are performed.

We next come to (2), i.e, $\mu\tau$ symmetry [4]–[30]. For the purpose of implementing it, we find it convenient to choose the following representation of the PMNS mixing matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ -s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}, \tag{1.4}$$

with $c_{ij} = cos\,\theta_{ij}$, $s_{ij} = sin\,\theta_{ij}$ δ being the Dirac phase and α, β being the Majorana phases. It is important to note that, with three real neutrino mass eigenvalues $m_{1,2,3}$, one has

$$m_\nu = U \text{ diag.}(m_1, m_2, m_3)\ U^T. \tag{1.5}$$

We can now define $\mu\tau$ symmetry to be the invariance of all couplings and masses in the pure neutrino part[3] of the Lagrangian under the interchange of the flavor indices 2 and 3. As a result,

$$(Y_\nu)_{12} = (Y_\nu)_{13}, \tag{1.6}$$
$$(Y_\nu)_{21} = (Y_\nu)_{31}, \tag{1.7}$$

[3]This symmetry is, of course, badly broken in the charged lepton sector.

$$(Y_\nu)_{23} = (Y_\nu)_{32}, \tag{1.8}$$
$$(Y_\nu)_{22} = (Y_\nu)_{33} \tag{1.9}$$

and

$$M_2 = M_3. \tag{1.10}$$

Using eq. (1.1), one then obtains

$$(m_\nu)_{12} = (m_\nu)_{13}, \tag{1.11}$$

$$(m_\nu)_{22} = (m_\nu)_{33}. \tag{1.12}$$

We shall take eqs.(1.11, 1.12) as the statement of a custodial $\mu\tau$ symmetry of the light neutrino mass matrix m_ν. An automatic consequence of these two equations is the fixing of the two mixing angles involving the third flavor at $\theta_{23} = \pi/4$, $\theta_{13} = 0$. Discarding unnatural cancellations, *sixty eight* of the *seventy two* allowed four zero textures in Y_ν are found to be incompatible with eqs.(1.11, 1.12) plus observational constraints. In particular, fifty two textures of category (A) and sixteen textures of (B) category are excluded. The two surviving textures of category A both lead to the same light neutrino mass matrix with $(m_\nu)_{23} = 0$. On the other hand, each of the two surviving category (B) textures turns out to have two zeroes in the first row and one each in the other rows and they also lead to the same light neutrino mass matrix. For each surviving texture, m_ν can be described, apart from an overall neutrino mass scale, by two real parameters and one phase angle, though their definitions are different for category A and category B. We call them k_1, k_2 and α for the former and l_1, l_2 and β for the latter. Their allowed ranges are found to be severely constrained by the neutrino oscillation data.

We then turn to the tribimaximal mixing (TBM) pattern [31]–[34] which implies $\theta_{13} = 0$, $\theta_{23} = \pi/4$ *and* $\theta_{12} = \sin^{-1}/\sqrt{3}$. The effect of $\mu\tau$ symmetry is thus subsumed here, but there is an additional constraint on θ_{12}. Hence all configurations of m_ν leading to TBM have not only to obey eqs.(1.11-1.12) but also the extra requirement

$$(m_\nu)_{11} + (m_\nu)_{13} = (m_\nu)_{22} + (m_\nu)_{23}. \tag{1.13}$$

The four textures of Y_ν, allowed by $\mu\tau$ symmetry, survive the imposition of eq. (1.13), but two relations between k_1, k_2 and α for category A and two between l_1, l_2 and β for category B emerge. Consequently, one independent real parameter k_2 for the former and one l_1 for the latter suffice to describe those textures after factoring out the overall mass scale. The allowed domains of k_2 and l_1 are again found to be highly restricted.

A general nondiagonal Majorana mass matrix m_ν in flavor space implies lepton flavor violation as well as the nonconservation of lepton number. It is therefore interesting and important to discuss the implications of the above forms of m_ν for[4] radiative lepton flavor violating decays ($\tau \to \mu\gamma,\ \tau \to e\gamma,\ \mu \to e\gamma$) and for realistic leptogenesis of both flavor independent and flavor dependent varieties. The former are yet-to-be-observed processes [49]

[4]Nonradiative lepton flavor violating processes, such as μe conversion in nuclei and triple charged leptonic decays of the τ and the μ, are not considered here since current experimental limits on those yield considerably weaker constraints than radiative lepton flavor violating decays.

for which the experimental sensitivity is rapidly approaching theoretical expectations; the latter is a desirable theoretical goal [50] of any (high scale) seesaw-based model of light neutrino masses and mixing angles. In the mSUGRA version [51] of a supersymmetric scenario, the branching ratios for the three radiative lepton flavor violating decays in question have rather simple flavor structures that are bilinear in Y_ν or m_D. We are thus able to make some specific predictions for our allowed textures, namely, the vanishing of BR($\tau \to \mu\gamma$) for category A and the value of the ratio BR($\tau \to e\gamma$)/BR($\mu \to e\gamma$) being $\simeq 0.178$ for both categories. Concerning leptogenesis, the term contributing only to flavor dependent lepton asymmetries vanishes for all flavor combinations in both categories. Regarding the term, which contributes to the flavorsummed lepton asymmetry, only the electron asymmetry gets generated in category A whereas the same always vanishes in category B. One can also make more definitive statements on specific flavor combinations of the latter term as well as on the effective mass for the washout of a particular flavor asymmetry.

One issue with $\mu\tau$ symmetry and TBM is that the former fixes θ_{13} and θ_{23} at 0 and $\pi/4$ respectively, while the latter further fixes θ_{12} at $\sin^{-1}\frac{1}{\sqrt{3}} \simeq 35.26°$. Though these numbers lie within presently allowed 3σ ranges of those mixing angles, the true values of the latter may eventually turn out to be different. There are, in fact, hints already that such may be the case. Current best fit 1σ ranges for those angles, derived from global analyses of all neutrino oscillation data, are [52] $\theta_{12} = 34.5° \pm 1.4°$, $\theta_{23} = 43.1°^{+4.4°}_{-3.5°}$ and $\theta_{13} = 8° \pm 2°$. While it is premature to take these ranges too seriously, it is nonetheless interesting to consider deviations within a definitive theoretical framework by taking them to originate dynamically from radiative effects. We impose $\mu\tau$ symmetry or TBM on elements of the light neutrino mass matrix m_ν at a high scale of the order of the lowest heavy right chiral neutrino mass, i.e. at $\Lambda \sim \min(M_1, M_2, M_3) \sim 10^{12}$ GeV. We further assume the validity of the Minimal Supersymmetric Standard Model (MSSM) [51] between this scale and the laboratory energy scale $\lambda \sim 10^3$ GeV. The elements of m_ν are then evolved from Λ to λ by one loop renormalization group running. Small deviations from the consequences of $\mu\tau$ symmetry or TBM, proportional to the square of the heaviest charged lepton mass divided by the Higgs VEV squared, are found to be generated. These lead to small but distinct extensions of the allowed values of $k_{1,2}$ in category A and $l_{1,2}$ in category B. Constrained deviations in the mixing angles also emerge.

The rest of the paper is organized as follows. Section 2 contains a discussion of the allowed four zero textures and their parameterization as a consequence of $\mu\tau$ symmetry and TBM. Radiative lepton flavor violating decays and leptogenesis are taken up for those textures in section 3. In section 4, radiatively induced small deviations in m_ν and their effects are discussed. The final section 5 contains a summary of our results and the conclusions derived therefrom. The appendix contains analytical expressions for the experimentally measured quantities utilized by us both without and with one loop RG evolution.

2 Allowed four zero textures

Category A. It is straightforward to see that only two of the fifty two four zero textures of category (A) are consistent with $\mu\tau$ symmetry, as implemented through eqs. (1.11, 1.2). The rest develop additional zeroes which are incompatible with known observational constraints and the assumption of no massless neutrino. The two allowed textures for the Dirac mass matrix $m_D = Y_\nu v$ can be given in terms of three complex parameters a_1, a_2, b_1 as

$$m_D^{(1)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_1 \\ 0 & b_1 & 0 \end{pmatrix}, \tag{2.1}$$

$$m_D^{(2)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_1 & 0 \\ 0 & 0 & b_1 \end{pmatrix}. \tag{2.2}$$

The corresponding light neutrino mass matrices are identical and can be written as

$$m_\nu^{(A)} = -\begin{pmatrix} a_1^2/M_1 + 2a_2^2/M_2 & a_2 b_1/M_2 & a_2 b_1/M_2 \\ a_2 b_1/M_2 & b_1^2/M_2 & 0 \\ a_2 b_1/M_2 & 0 & b_1^2/M_2 \end{pmatrix}. \tag{2.3}$$

Let us now define $m \equiv -\frac{b_1^2}{M_2}$, $k_1 e^{i(\alpha+\alpha')} \equiv \frac{a_1}{b_1}\frac{\sqrt{M_2}}{\sqrt{M_1}}$, $k_2 e^{i\alpha'} \equiv \frac{a_2}{b_1}$ and further absorb the phase α' in the first family neutrino field ν_e. The latter is equivalent to rotating the mass matrix of eq. (2.3) by the phase matrix diag.$(e^{-i\alpha'}, 1, 1)$. This operation changes eq. (2.3) to

$$m_\nu^{(A)} = m \begin{pmatrix} k_1^2 e^{2i\alpha} + 2k_2^2 & k_2 & k_2 \\ k_2 & 1 & 0 \\ k_2 & 0 & 1 \end{pmatrix}. \tag{2.4}$$

Apart from the overall mass scale factor m, the light neutrino mass matrix now has two real parameters k_1, k_2 and the phase angle α.

The ratio $R = \Delta m_{21}^2/\Delta m_{32}^2$ and the solar/reactor mixing angle θ_{12} are now given by

$$R = 2(X_1^2 + X_2^2)^{1/2}\left[X_3 - (X_1^2 + X_2^2)^{1/2}\right]^{-1}, \tag{2.5}$$

$$\tan 2\theta_{12} = \frac{X_1}{X_2} \tag{2.6}$$

with

$$X_1 = 2\sqrt{2}k_2\left[(1+2k_2^2)^2 + k_1^4 + 2k_1^2(1+2k_2^2)\cos 2\alpha\right]^{1/2}, \tag{2.7}$$

$$X_2 = 1 - k_1^4 - 4k_2^4 - 4k_1^2 k_2^2 \cos 2\alpha, \tag{2.8}$$

$$X_3 = 1 - 4k_2^4 - k_1^4 - 4k_1^2 k_2^2 \cos 2\alpha - 4k_2^2. \tag{2.9}$$

The observables of eqs.(2.5) and (2.6) can be compared with the available data. We see right away that the expression for R is *incompatible with a normal mass ordering ($\Delta m_{32}^2 > 0$) and can only accommodate an inverted one ($\Delta m_{32}^2 < 0$)*. This is consistent with the conclusion of Merle and Rodejohann [53] who had shown that the condition $(m_\nu)_{23} = (m_\nu)_{32} = 0$ is compatible only with an inverted mass ordering. The allowed ranges are given respectively[5] by $R = -3.476 \times 10^{-2}$ eV^2 to -2.972×10^{-2} eV^2 at the 1σ level and -4.129×10^{-2} eV^2 to

[5]We are using the range of R extracted [54] by assuming an inverted mass-ordering.

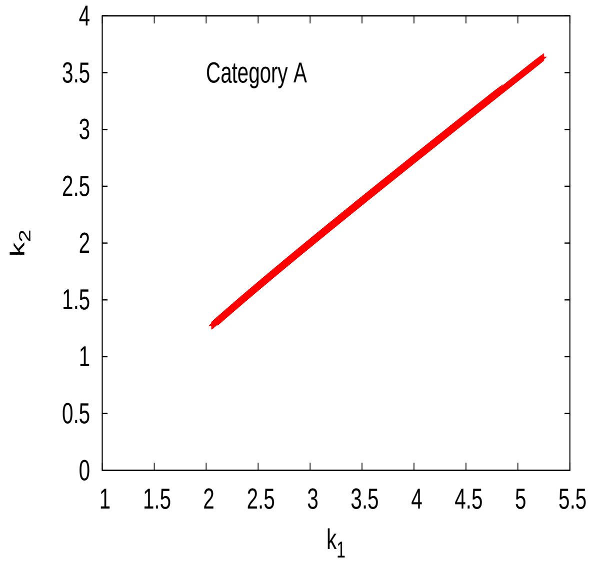

Figure 1. Variation of k_1 and k_2 in category A with $\mu\tau$ symmetry over the 3σ allowed ranges of R and θ_{12}.

-2.534×10^{-2} eV^2 at the 3σ level and by $\tan 2\theta_{12} = 3.045$ - 2.278 at 1σ and 4.899 - 1.828 at 3σ. The angle α is immediately found to be correspondingly restricted to be between 89° and 90°. We find that *there is no acceptable solution for the 1σ-allowed range of R.* For the 3σ-allowed range of the latter, a very narrow strip is allowed in the k_1-k_2 plane for the allowed domain of α, as shown in figure 1 with $2.0 < k_1 < 5.3$ and $1.2 < k_2 < 3.7$. Thus $m_\nu^{(A)}$ may quite possibly be excluded by further improvements of error in the data on R and $\tan 2\theta_{12}$.

On further assuming tribimaximal neutrino mixing, i.e, eq. (1.13), one obtains the relation

$$k_1^2 e^{2i\alpha} + 2k_2^2 + k_2 = 1. \tag{2.10}$$

Given eq. (2.10), α is now fixed[6] to be $\pi/2$ and the two real parameters $k_{1,2}$ are therefore reduced to one, which we take to be k_2 fixing k_1 at

$$k_1 = \left(2k_2^2 + k_2 - 1\right)^{1/2}. \tag{2.11}$$

Now that $\tan 2\theta_{12}$ is fixed at $2\sqrt{2}$, the ratio R is given by

$$R = \frac{3(k_2 - 2)}{k_2 + 2}. \tag{2.12}$$

The range of k_2 restricted by the 3σ allowed domain of R is now $1.95 \le k_2 \le 1.97$ so that its value is fixed to the first decimal place.

[6]The solution $\alpha = 0$ is incompatible with the allowed range of R and the reality of $k_{1,2}$.

Category B. Again, in this case, only two of the original eighteen textures are allowed by $\mu\tau$ symmetry. These may be written in terms of three complex parameters a_1, b_1, b_2 as

$$m_D^{(3)} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}, \tag{2.13}$$

$$m_D^{(4)} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}, \tag{2.14}$$

with the corresponding light neutrino mass matrices both being

$$m_\nu^{(B)} = -\begin{pmatrix} a_1^2/M_1 & a_1 b_1/M_1 & a_1 b_1/M_1 \\ a_1 b_1/M_1 & b_1^2/M_1 + b_2^2/M_2 & b_1^2/M_1 \\ a_1 b_1/M_1 & b_1^2/M_1 & b_1^2/M_1 + b_2^2/M_2 \end{pmatrix}. \tag{2.15}$$

Now, we choose to define $m = -\frac{b_2^2}{M_2}$, $l_1 e^{i\beta'} = \frac{a_1}{b_2}\frac{\sqrt{M_2}}{\sqrt{M_1}}$, $l_2 e^{i\beta} = \frac{b_1}{b_2}\frac{\sqrt{M_2}}{\sqrt{M_1}}$ and absorb the phase β' in ν_e. We are then left with

$$m_\nu^{(B)} = m \begin{pmatrix} l_1^2 & l_1 l_2 e^{i\beta} & l_1 l_2 e^{i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} + 1 & l_2^2 e^{2i\beta} \\ l_1 l_2 e^{i\beta} & l_2^2 e^{2i\beta} & l_2^2 e^{2i\beta} + 1 \end{pmatrix}. \tag{2.16}$$

The measurable quantities R and $\tan 2\theta_{12}$ are still given by eqs.(2.5) and (2.6), but now the functions $X_{1,2,3}$ are given in terms of the parameters (l_1, l_2 , β) as

$$X_1 = 2\sqrt{2} l_1 l_2 \left[(l_1^2 + 2l_2^2)^2 + 1 + 2(l_1^2 + 2l_2^2)\cos 2\beta\right]^{1/2}, \tag{2.17}$$

$$X_2 = 1 + 4l_2^2 \cos 2\beta + 4l_2^4 - l_1^4, \tag{2.18}$$

$$X_3 = 1 - (l_1^2 + 2l_2^2)^2 - 4l_2^2 \cos 2\beta. \tag{2.19}$$

In this case we see that the expression for R *admits only a normal mass ordering and disallows an inverted one*. A comparison with data fixes β in the ranges 87° to 90° and 89° to 90° respectively for the values of[7] $R = 3.329 \times 10^{-2}$ eV^2 to 2.858×10^{-2} eV^2 at the 1σ level and 3.915×10^{-2} eV^2 to 2.455×10^{-2} eV^2 at the 3σ level with the allowed values of $\tan 2\theta_{12}$ as previously mentioned. The corresponding allowed values of $l_{1,2}$ are shown in figure 2 for the 3σ-allowed range. Unlike category A, a substantial region of the parameter space, consisting of two branches, is allowed here.

The imposition of the tribimaximal mixing condition of eq. (1.13) now leads to

$$l_1^2 + l_1 l_2 e^{i\beta} = 2l_2^2 e^{2i\beta} + 1 \tag{2.20}$$

which fixes β by[8]

$$\cos\beta = \frac{l_1}{4l_2}. \tag{2.21}$$

[7] We are using the range of R extracted [54] by assuming a normal mass-ordering.
[8] The solution $\beta = 0$ is not compatible with real $l_{1,2}$ and the allowed range of R.

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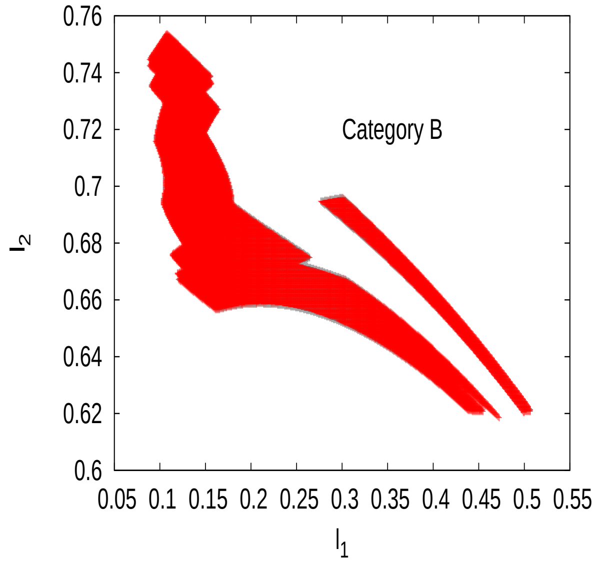

Figure 2. Variation of l_1 and l_2 in category B with $\mu\tau$ symmetry over the 3σ allowed ranges of R and θ_{12}.

Moreover, $l_{1,2}$ can now be reduced to a single real parameter l_1 with l_2 given by

$$l_2 = \frac{1}{2}(1 - l_1^2)^{1/2}. \tag{2.22}$$

Again, $\tan 2\theta_{12}$ being $2\sqrt{2}$, R is given by

$$R = \frac{3l_1^2}{2 - 4l_1^2}. \tag{2.23}$$

In consequence, the allowed 1σ and 3σ ranges of l_1 get restricted to $0.12 \leq l_1 \leq 0.13$ and $0.11 \leq l_1 \leq 0.15$ respectively. Once again, the value of this surviving one parameter is fixed to the first decimal place.

3 Radiative lepton flavor violation and leptogenesis

Radiative lepton flavor violating decays $l_\alpha \to l_\beta \gamma$ (flavor indices α, β spanning $1 = e, 2 = \mu, 3 = \tau$ with the constraint $\alpha > \beta$) together with the required generation of a lepton asymmetry at a high scale, provide powerful tools to check and test [55]–[73] any proposed seesaw-based scheme of neutrino mixing and masses. There already exist lower bounds on the partial lifetimes of the former processes; moreover, forthcoming experiments with higher sensitivity will hope to observe some of the decay channels. Coming to leptogenesis as a route to baryogenesis, a fair amount of theoretical understanding exists for high scale leptogenesis - both of the flavored and unflavored varieties. In this section, we explore the implications of the allowed four zero texture configurations, with tribimaximal mixing or at least $\mu\tau$ symmetry, for these two types of phenomena.

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We note first the one-loop expression [74] for BR($l_\alpha \to l_\beta \gamma$) which is valid in mSUGRA scenarios with universal boundary conditions on the masses of scalar particles at a high scale M_X:

$$\mathrm{BR}(l_\alpha \to l_\beta \gamma) = \mathrm{const.}\, \mathrm{BR}(l_\alpha \to l_\beta \nu \bar{\nu}) |(m_D L m_D^\dagger)_{\alpha\beta}| \tag{3.1}$$

with

$$L_{kl} = \ln \frac{M_X}{M_k} \delta_{kl}, \tag{3.2}$$

M_k being the mass of the kth. heavy right chiral neutrino. The matrix L takes care of the RG running from M_X to M_k. We can now discuss what happens with our four allowed configurations for m_D.

Category A. For both the allowed textures $m_D^{(1)}$ and $m_D^{(2)}$, we have

$$(m_\nu)_{23} = -(m_D M_R^{-1} m_D^T)_{23} = 0 \tag{3.3}$$

in a basis in which M_R is diagonal. Since L is a diagonal matrix, it follows that

$$(m_D L m_D^\dagger)_{23} = 0. \tag{3.4}$$

Consequently,

$$\mathrm{BR}(\tau \to \mu\gamma) = 0. \tag{3.5}$$

and any observation of the $\tau \to \mu\gamma$ process will rule out these configurations. It has moreover been shown [53] from the twin requirements of two nonzero neutrino masses and mixing angles that in such a case $(m_\nu)_{12} \neq 0 \neq (m_\nu)_{13}$. As a result, $(m_D L m_D^\dagger)_{12}$ and $(m_D L m_D^\dagger)_{13}$ are also both nonzero, leading to nonvanishing rates for the decays $\mu \to e\gamma$ and $\tau \to e\gamma$ respectively. There is moreover a relation between them. On account of $\mu\tau$ symmetry, $M_2 = M_3$ and $(m_D L m_D^\dagger)_{12} = (m_D L m_D^\dagger)_{13}$, so that we have

$$\frac{\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\mu \to e\gamma)} \simeq \frac{\mathrm{BR}(\tau \to \mathrm{e}\nu_\mathrm{e}\bar{\nu}_\mathrm{e})}{\mathrm{BR}(\mu \to \mathrm{e}\nu_\mu\bar{\nu}_\mu)} \simeq 0.178. \tag{3.6}$$

Category B. For both the allowed textures $m_D^{(3)}$ and $m_D^{(4)}$, the matrix $m_D L m_D^\dagger$ is identical with all elements nonvanishing. Thus, all the three radiative modes $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$ are possible. However, $\mu\tau$ symmetry has the same consequence as in category A, i.e eq. (3.6) holds here too.

We next turn to leptogenesis at the scale $\sim \min(M_1, M_2, M_3)$ which for simplicity we take to be M_1. Most pertinent for this are the lepton asymmetries generated by the decay of a heavy right chiral neutrino N_i into a lepton of flavor α (= e, μ, τ) and a Higgs ϕ:

$$\begin{aligned}\epsilon_i^\alpha &= \frac{\Gamma(N_i \to \phi \bar{l}_\alpha) - \Gamma(N_i \to \phi^\dagger l_\alpha)}{\sum_\beta [\Gamma(N_i \to \phi \bar{l}_\beta) + \Gamma(N_i \to \phi^\dagger l_\beta)]} \\ &\simeq \frac{g^2}{16\pi M_W^2} \frac{1}{(m_D^\dagger m_D)_{ii}} \sum_{j \neq i} \left[\mathcal{I}_{ij}^\alpha f\left(\frac{M_j^2}{M_i^2}\right) + \mathcal{J}_{ij}^\alpha f\left(1 - \frac{M_j^2}{M_i^2}\right)^{-1} \right]\end{aligned} \tag{3.7}$$

configuration	$\mathcal{I}_{ij}^{\alpha}$	$\mathcal{J}_{ij}^{\alpha}$	$\widetilde{m_1}^e$	$\widetilde{m_1}^\mu$	$\widetilde{m_1}^\tau$
$m_D^{(1)}$	$\mathcal{I}_{12}^e = \mathcal{I}_{13}^e \neq 0$, rest zero	0	nonzero	0	0
$m_D^{(2)}$	–do–	0	nonzero	0	0
$m_D^{(3)}$	$\mathcal{I}_{12}^\mu = \mathcal{I}_{13}^\tau \neq 0$, rest zero	0	nonzero	nonzero	equals $\widetilde{m_1}^\mu$
$m_D^{(4)}$	$\mathcal{I}_{13}^\mu = \mathcal{I}_{12}^\tau \neq 0$, rest zero	0	nonzero	nonzero	equals $\widetilde{m_1}^\mu$

Table 1. Leptogenesis table

where we have neglected $O(M_W^2/M_i^2)$ terms. Here

$$\mathcal{I}_{ij}^{\alpha} = \mathrm{Im}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ij} = -\mathcal{I}_{ji}^{\alpha},$$

$$\mathcal{J}_{ij}^{\alpha} = \mathrm{Im}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}(m_D^\dagger m_D)_{ji} = -\mathcal{J}_{ji}^{\alpha}. \tag{3.8}$$

The function $f(x)$ has the form

$$f(x) = \sqrt{x}\left[\frac{2}{1-x} - \ln\frac{1+x}{x}\right] \tag{3.9}$$

in the MSSM. For $M_1 \ll M_{2,3}$, $f(M_{2,3}^2/M_1^2) \simeq -3M_1/M_{2,3}$ in which case the $\mathcal{J}_{ij}^{\alpha}$ term in ϵ_i^α gets suppressed by $M_1/M_{2,3}$. Another interesting quantity is the effective mass for the washout of a flavor asymmetry. This is given by [75]–[77]

$$\tilde{m}_1^\alpha = |(m_D)_{\alpha 1}|^2/M_1 \tag{3.10}$$

and controls the magnitude of the final baryon asymmetry Y_B in the way shown in ref. [75]–[77]. Summing over all lepton flavors α, the $\mathcal{J}_\alpha^{ij}$ term drops out since $\sum_\alpha \mathcal{J}_{ij}^{\alpha} = 0$. Utilizing the result that $\mathcal{I}_{ij} = \sum_\alpha \mathcal{I}_{ij}^{\alpha} = \mathrm{Im}[(m_D^\dagger m_D)_{ij}]^2$, we have

$$\epsilon_i = \sum_\alpha \epsilon_i^\alpha = \frac{g^2}{16\pi M_W^2}\frac{1}{(m_D^\dagger m_D)_{ii}}\sum_{j\neq i}[(m_D^\dagger m_D)_{ij}]^2 f\left(M_j^2/M_i^2\right). \tag{3.11}$$

Though the above expressions are valid in the MSSM, their flavor structure is just that of the Standard Model.

Selecting the $\mu\tau$ symmetric four zero texture configurations of m_D, we find that $\mathcal{J}_{ij}^{\alpha}$ vanishes in every case for all α, i, j. Thus we need not consider the second term in eq. (3.7) at all. Regarding $\mathcal{I}_{ij}^{\alpha}$, both allowed textures in category A yield the same result, namely, $\mathcal{I}_{12}^e = \mathcal{I}_{13}^e \neq 0$ while the other combinations vanish. Therefore, only the electron asymmetry gets generated in this case. In category B, $m_D^{(3)}$ leads to nonzero and equal $\mathcal{I}_{12}^\mu$, $\mathcal{I}_{13}^\tau$, with all other $\mathcal{I}_{ij}^{\alpha}$ vanishing, while $m_D^{(4)}$ yields nonvanishing and equal $\mathcal{I}_{13}^\mu$, $\mathcal{I}_{12}^\tau$, the rest of $\mathcal{I}_{ij}^{\alpha}$ being zero. Turning to the effective washout mass, only the electron one, namely $\widetilde{m_1}^e$, is nonvanishing for both textures of category A. For those of category B, all the washout masses $\widetilde{m_1}^e$, $\widetilde{m_1}^\mu$, $\widetilde{m_1}^\tau$ are nonzero with $\widetilde{m_1}^\mu = \widetilde{m_1}^\tau$. We provide a table containing the relevant information on leptogenesis parameters for each of our allowed four texture zero configurations.

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4 Radiatively induced deviations

We mentioned in the previous section that the results $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ follow from a custodial $\mu\tau$ symmetry in m_ν. A breaking of this symmetry would in general result in a nonzero value of θ_{13} as well as a departure of θ_{23} from $\pi/4$. The goals of many ongoing and planned experiments are to measure their actual values [78]. Another interesting consequence of a nonzero θ_{13} would be the presence of a CKM-type of CP violation in the lepton sector. Our previous expressions for R and $\tan 2\theta_{12}$ will be modified if $\mu\tau$ symmetry is indeed broken.

In this section we invoke the dynamical origin of such a symmetry breaking due to the Renormalization Group (RG) evolution of the elements of the neutrino mass matrix. Our basic idea is to posit that $\mu\tau$ symmetry (or more restrictively whichever symmetry, say A_4 or S_3 is responsible for TBM) is valid at a high energy scale $\Lambda \sim 10^{12}$ GeV which characterizes the heavy right chiral neutrinos N_i. We then consider the radiative breaking of such a symmetry through charged lepton mass terms, induced at the one loop level, as one evolves by RG running to the lower energy scale $\lambda \sim 10^3$ GeV. The specific theory in which we choose to do this is the Minimal Supersymmetric Standard Model (MSSM) [51] with an intrasupermultiplet mass splitting, caused by explicit supersymmetry breaking, being $O(\lambda)$. Following the methodology described in ref. [79]–[80], we consider the neutrino mass matrices $m_n u$ given in eqs.(2.4) and (2.16) at the high scale Λ. Their evolved form at the low scale λ is then given approximately by[9]

$$m_\nu^\lambda \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\Delta_\tau \end{pmatrix} \; m_\nu \; \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\Delta_\tau \end{pmatrix}. \tag{4.1}$$

The proportionality involves a scale factor which is not relevant to our present analysis. The factor Δ_τ is due to one loop RG evolution and we can neglect m_e^2 and m_μ^2 terms as compared to m_τ^2. Δ_τ is given approximately by

$$\Delta_\tau \simeq \frac{m_\tau^2}{8\pi^2 v^2}(\tan^2\beta + 1)\ln\left(\frac{\Lambda}{\lambda}\right), \tag{4.2}$$

where $\tan\beta$ is the ratio of the VEVs of the up-type and down-type neutral Higgs fields in the MSSM and v^2 is twice the sum of their squares. Suppose the $\mu\tau$ symmetric form of m_ν is written as

$$m_\nu = m \begin{pmatrix} P & Q & Q \\ Q & R & S \\ Q & S & R \end{pmatrix}, \tag{4.3}$$

where the complex quantities P, Q, R, S are to be identified from the neutrino mass matrices given in eq. (2.4) or (2.16). Then the corresponding neutrino mass matrix at the low energy scale λ comes out as

$$m_\nu^\lambda = m \begin{pmatrix} P & Q & Q(1-\Delta_\tau) \\ Q & R & S(1-\Delta_\tau) \\ Q(1-\Delta_\tau) & Q(1-\Delta_\tau) & R(1-2\Delta_\tau) \end{pmatrix}. \tag{4.4}$$

[9] In terms of Y_ν with which we started, $Y_\nu^\lambda \simeq \text{diag.}(1,1,1-\Delta_\tau)Y_\nu$.

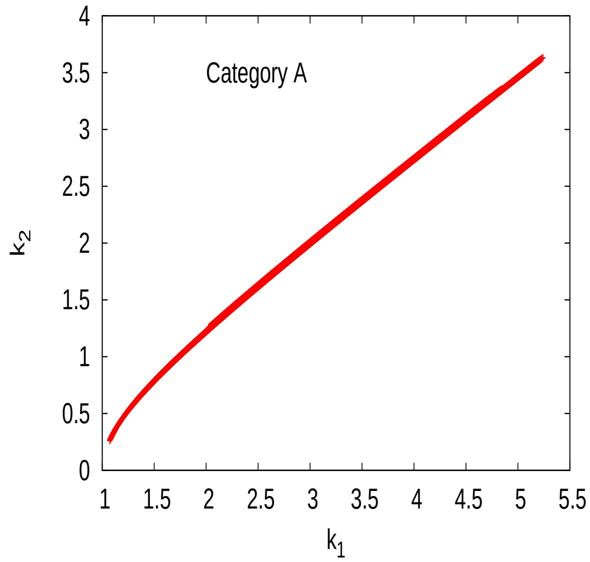

Figure 3. The allowed variation of k_1 vs k_2 including radiative deviation within 3σ allowed ranges of R^λ and θ_{12}^λ. The phase angle α does not change significantly to $O(\Delta_\tau)$.

From eq. (4.4) we can calculate R^λ as well as $\sin\theta_{12}^\lambda$, $\sin\theta_{23}^\lambda$ and $\sin\theta_{13}^\lambda$ for the allowed textures of category A and category B. The corresponding analytic expressions are given in the appendix. There is now a slight extension of the allowed regions in the k_1-k_2 plane for category A and in the l_1-l_2 plane for category B are shown in figures 3 and 4 respectively. For the allowed category A textures, we find that *any value of* θ_{23}^λ *greater than* 45° *is disallowed*. Then the experimentally allowed 3σ ranges $30.7^\circ \leq \theta_{12}^\lambda \leq 39.2^\circ$, $36^\circ \leq \theta_{23}^\lambda \leq 45^\circ$ and the maximum allowed value $\simeq 60$ of $\tan\beta$ [51] restrict θ_{13}^λ to $0^\circ \leq \theta_{13}^\lambda \leq 2.7^\circ$. Similarly, for the allowed category B textures, we find that *any value of* θ_{23}^λ *less than* 45° *is excluded*. For the 3σ allowed ranges $45^\circ \leq \theta_{23}^\lambda \leq 54^\circ$ and $30.7^\circ \leq \theta_{12}^\lambda \leq 39.2^\circ$, θ_{13}^λ is found to be in the interval $0^\circ \leq \theta_{13}^\lambda \leq 0.85^\circ$.

5 Concluding summary

This paper has investigated the effect of $\mu\tau$ symmetry and (more restrictively) TBM on the maximally allowed four zero neutrino Yukawa textures within the type I seesaw in the weak basis where charged leptons and the three heavy right chiral neutrino are mass diagonal. Only two textures (leading to the same from of m_ν) out of fifty four in category A and two textures (again leading to an identical m_ν form) out of eighteen in category B survive the imposition of $\mu\tau$ symmetry. Each m_ν can be characterized by two real parameters and one phase: chosen to be k_1, k_2, α for category A and l_1, l_2, β for category B. All are severely constrained by extant neutrino oscillation data. In each category, the additional requirement of TBM reduces the three parameters to a single real constant with a nearly fixed value.

We have further looked at radiative lepton flavor violating decays $l_\alpha \rightarrow l_\beta\gamma$ (with

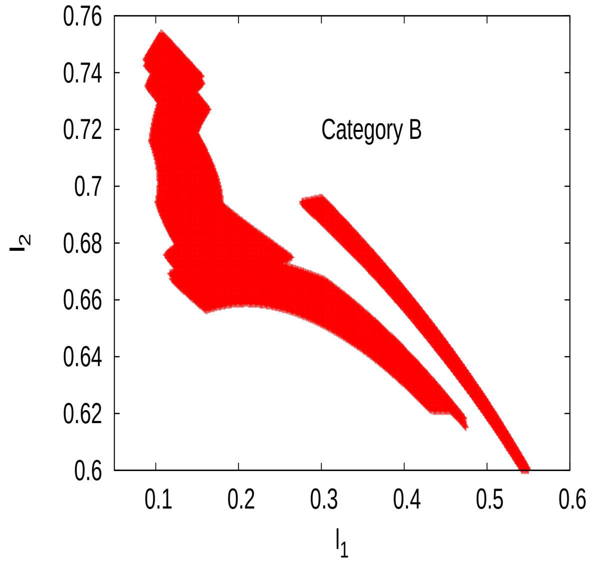

Figure 4. The allowed variation of l_1 vs l_2 including radiative deviation within 3σ allowed ranges of R^λ and θ^λ_{12}. The phase angle β does not change significantly to $O(\Delta_\tau)$.

$\alpha > \beta$=1,2,3) in the mSUGRA version of the MSSM. Our conclusion is that BR$(\tau \to \mu\gamma) = 0$ for category A and BR$(\tau \to e\gamma)$/BR$(\mu \to e\gamma) \simeq 0.178$ for both categories. Leptogenesis has also been considered at the energy scale min.(M_1, M_2, M_3) with the following result. The term $\mathcal{J}^\alpha_{ij}$, which does not contribute to the flavor-summed lepton asymmetry, vanishes in either category. The term $\mathcal{I}^\alpha_{ij}$, which can cause such an asymmetry, is constrained. In particular, (1) $\mathcal{I}^e_{12}$ and $\mathcal{I}^e_{13}$ are nonzero while the other contributions vanish in category A; (2) either $\mathcal{I}^\mu_{12}$, $\mathcal{I}^\tau_{13}$ or $\mathcal{I}^\mu_{13}$, $\mathcal{I}^\tau_{12}$ are nonzero with the rest vanishing in category B. Regarding effective washout masses, only $\tilde{m^e_1}$ is nonvanishing in category A, while all of $\tilde{m^e_1}$, $\tilde{m^\mu_1}$, $\tilde{m^\tau_1}$ are nonzero in category B with $\tilde{m^\mu_1} = \tilde{m^\tau_1}$.

Finally, deviations from $\mu\tau$ symmetry, that are radiative in origin, have been considered. First, this symmetry has been imposed on m_ν at $\Lambda \sim 10^{12}$ GeV which typifies an energy scale that is characteristic of the heavy right chiral neutrino masses. Then the deviations in the elements of m_ν, caused by one-loop RG running from Λ to the laboratory scale $\lambda \sim 10^3$ GeV, have been computed in the MSSM with the largest allowed value of $\tan\beta$. Using the experimental 3σ ranges of R and θ_{12}, we have found the following results:

(1) category A allows only an inverted neutrino mass ordering ($\Delta m^2_{32} < 0$) with $\theta_{23} \leq 45°$ and $0° \leq \theta_{13} \leq 2.7°$;

(2) only a normal mass ordering ($\Delta m^2_{32} > 0$) with $\theta_{23} \geq 45°$ and $0° \leq \theta_{13} \leq 0.85°$ are allowed in category B.

These predictions will face crucial future tests of the allowed four zero neutrino Yukawa textures in our scenario. Our bottom line is that $m_\nu^{(A)}$ is on the verge of exclusion, while

$m_\nu^{(B)}$ is a good candidate for the true m_ν occurring in nature. A measured value of θ_{13} will provide a crucial test of latters viability.

A Expressions for measurable quantities

$\mu\tau$ symmetric case. Eq. (4.3) leads to

$$h = m_\nu m_\nu^\dagger = m^2 \begin{pmatrix} |P|^2 + 2|Q|^2 & PQ^\star + Q(R^\star + S^\star) & PQ^\star + Q(R^\star + S^\star) \\ P^\star Q + Q^\star(R+S) & |Q|^2 + |R|^2 + |S|^2 & |Q|^2 + RS^\star + R^\star S \\ P^\star Q + Q^\star(R+S) & |Q|^2 + R^* S + RS^* & |Q|^2 + |R|^2 + |S|^2 \end{pmatrix}. \tag{A.1}$$

The diagonalization of h yields diag.(m_1^2, m_2^2, m_3^2) and also expressions for five relevant measurable quantities. The latter are:

(1) $\Delta m_{21}^2 = m_2^2 - m_1^2$, i.e the light neutrino mass squared difference relevant to solar/reactor experiments,

(2) the corresponding mixing angle θ_{12},

(3) $\Delta m_{32}^2 = m_3^2 - m_2^2$, i.e the neutrino mass squared difference pertaining to atmospheric/long-baseline studies,

(4) the corresponding mixing angle θ_{23} and

(5) the remaining mixing angle θ_{13}.

The last five quantities can all be expressed in terms of three real functions $X_{1,2,3}$ of the complex quantities P, Q, R, S appearing in m_ν. These are defined as

$$\begin{aligned} X_1 &= 2\sqrt{2}|PQ^\star + Q(R^\star + S^\star)|, \\ X_2 &= |R+S|^2 - |P|^2, \\ X_3 &= |R+S|^2 - |P|^2 - 4(|Q|^2 + RS^\star + R^\star S). \end{aligned} \tag{A.2}$$

We then have

$$\Delta m_{21}^2 = m^2 (X_1^2 + X_2^2)^{1/2}, \tag{A.3}$$

$$\theta_{12} = \frac{1}{2}\tan^{-1}\frac{X_1}{X_2}, \tag{A.4}$$

$$\Delta m_{32}^2 = \frac{m^2}{2}[X_3 - (X_1^2 + X_2^2)^{1/2}], \tag{A.5}$$

$$\theta_{23} = \frac{\pi}{4}, \tag{A.6}$$

$$\theta_{13} = 0. \tag{A.7}$$

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Case with RG- broken $\mu\tau$ symmetry. We work to one loop and ignore $O(\Delta_\tau^2)$ terms. Now from eq. (4.4) one derives that

$$m_\nu^\lambda m_\nu^{\lambda\dagger} = h - m^2\Delta_\tau \begin{pmatrix} 2|Q|^2 & 2QS^\star & PQ^\star + QS^\star + 3QR^\star \\ 2Q^\star S & 2|S|^2 & |Q|^2 + RS^\star + 3R^\star S \\ P^\star Q + Q^\star S + 3Q^\star R & |Q|^2 + R^\star S + 3RS^\star & 2(|Q|^2 + |S|^2) + 4|R|^2 \end{pmatrix}. \tag{A.8}$$

The cumbersome diagonalization of $m_\nu^\lambda m_\nu^{\lambda\dagger}$ is avoidable since the algebra simplifies in the specific cases of category A and category B. Let us reintroduce $c_{12} = \cos\theta_{12}$, $s_{12} = \sin\theta_{12}$ where θ_{12} is given in eq. (A.4). We now define five functions $F_{1,\ldots 5}$ in terms of c_{12}, s_{12} and elements of the m_ν matrix P, Q, R and S. The five functions $F$are

$$\begin{aligned}
F_1 &= -\sqrt{2}c_{12}^2 \frac{\{P^*Q + 3Q^*(R+S)\}\{PQ^\star + Q(R^\star + S^\star)\}}{|PQ^\star + Q(R^\star + S^\star)|} + 4c_{12}s_{12}\,|R+S|^2 \\
&\quad + \sqrt{2}s_{12}^2 \frac{\{PQ^* + 3Q(R^* + S^*)\}\{P^*Q + Q^*(R+S)\}}{|PQ^\star + Q(R^\star + S^\star)|}, \\
F_2 &= -\sqrt{2}c_{12} \frac{\{P^*Q + Q^*(3R - S)\}\{PQ^\star + Q(R^\star + S^\star)\}}{|PQ^\star + Q(R^\star + S^\star)|} + 2s_{12}\left(|Q|^2 + 2|R|^2 + RS^* - SR^*\right), \\
F_3 &= -4\sqrt{2}c_{12}s_{12} \frac{\left\{|PQ^\star + Q(R^\star + S^\star)|^2 + 2|Q|^2|R+S|^2 + Q^2P^*(R^* + S^*) + Q^{*2}P(R+S)\right\}}{|PQ^\star + Q(R^\star + S^\star)|} \\
&\quad -4(c_{12}^2 - s_{12}^2)\,|R+S|^2 \\
F_4 &= -\sqrt{2}s_{12} \frac{\{P^*Q + Q^*(3R - S)\}\{PQ^\star + Q(R^\star + S^\star)\}}{|PQ^\star + Q(R^\star + S^\star)|} - 2c_{12}\left(|Q|^2 + 2|R|^2 + RS^* - SR^*\right), \\
F_5 &= 2\sqrt{2}c_{12}s_{12} \frac{\left\{|PQ^\star + Q(R^\star + S^\star)|^2 + 2|Q|^2|R+S|^2 + Q^2P^*(R^* + S^*) + Q^{*2}P(R+S)\right\}}{|PQ^\star + Q(R^\star + S^\star)|} \\
&\quad -4\,|R-S|^2 + 4s_{12}^2|Q|^2 + 4c_{12}^2\left\{|Q|^2 + |R+S|^2\right\}.
\end{aligned} \tag{A.9}$$

Thus F_3 and F_5 are real, while F_1, F_2 and F_4 are in general complex. We now list the changed values of the earlier mentioned five measurable quantities.

$$(\Delta m_{21}^2)^\lambda = \Delta m_{21}^2 + \frac{1}{2}m^2 F_3 \Delta_\tau, \tag{A.10}$$

$$\theta_{12}^\lambda = \sin^{-1}|s_{12} + \frac{m^2 c_{12}}{2\Delta m_{21}^2}F_1^\star \Delta_\tau|, \tag{A.11}$$

$$(\Delta m_{32}^2)^\lambda = \Delta m_{32}^2 + \frac{1}{2}m^2 F_5 \Delta_\tau, \tag{A.12}$$

$$\theta_{23}^\lambda = \sin^{-1}\left|\frac{1}{\sqrt{2}} + \frac{\Delta_\tau}{2\sqrt{2}}m^2\left(\frac{s_{12}F_2^\star}{\Delta m_{21}^2 + \Delta m_{32}^2} - \frac{c_{12}F_4^\star}{\Delta m_{32}^2}\right)\right|, \tag{A.13}$$

$$\theta_{13}^\lambda = \frac{\Delta_\tau}{2\sqrt{2}}m^2\left|\frac{c_{12}F_2^\star}{\Delta m_{21}^2 + \Delta m_{32}^2} - \frac{s_{12}F_4^\star}{\Delta m_{32}^2}\right| \tag{A.14}$$

Note that, upto order Δ_τ, we can write the changed value of the ratio R as

$$R^\lambda = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} + \frac{1}{2}m^2\Delta_\tau\left(\frac{F_3}{\Delta m_{32}^2} - F_5\frac{\Delta m_{21}^2}{(\Delta m_{32}^2)^2}\right). \tag{A.15}$$

For convenience, we list the quantities P, Q, R and S in each category below:

Category A. From elements of $m_\nu^{(A)}$ in eq. (2.4)

$$\begin{aligned} P &= k_1^2 e^{2i\alpha} + 2k_2^2, \\ Q &= k_2, \\ R &= 1, \\ S &= 0. \end{aligned} \tag{A.16}$$

Category B. From elements of $m_\nu^{(B)}$ in eq. (2.16)

$$\begin{aligned} P &= l_1^2, \\ Q &= l_1 l_2 e^{i\beta}, \\ R &= l_2^2 e^{2i\beta} + 1, \\ S &= l_2^2 e^{2i\beta}. \end{aligned} \tag{A.17}$$

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Erratum

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1a) In eq. (1.4), the parameters 'α' and 'β' should be replaced by 'α_M' and 'β_M', respectively. The Eqn. should read as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ -s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_M} & 0 \\ 0 & 0 & e^{i(\beta_M+\delta)} \end{pmatrix} \quad (1.4)$$

1b) Next line of eq. (1.4) should read as " with $c_{ij} = cos\,\theta_{ij}$, $s_{ij} = sin\,\theta_{ij}$, δ being the Dirac phase and α_M, β_M being the Majorana phases".

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2a) The second line below eq. (2.19), should read as "A comparison with data fixes β in the ranges 89° to 90° and 87° to 90° respectively for the values of ......".

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3a) In eq. (3.6), numerator of the second term '$e\nu_e\bar{\nu}_e$' should be '$e\nu_\tau\bar{\nu}_e$' and the eq. should read as

$$\frac{\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\mu \to e\gamma)} \simeq \frac{\mathrm{BR}(\tau \to \mathrm{e}\nu_\tau\bar{\nu}_\mathrm{e})}{\mathrm{BR}(\mu \to \mathrm{e}\nu_\mu\bar{\nu}_\mu)} \simeq 0.178. \quad (3.6)$$

3b) In eq. (3.7), there should be no 'f' in the second term of second line and it should read as

$$\begin{aligned} \epsilon_i^\alpha &= \frac{\Gamma(N_i \to \phi\bar{l}_\alpha) - \Gamma(N_i \to \phi^\dagger l_\alpha)}{\sum_\beta[\Gamma(N_i \to \phi\bar{l_\beta}) + \Gamma(N_i \to \phi^\dagger l_\beta)]} \\ &\simeq \frac{g^2}{16\pi M_W^2}\frac{1}{(m_D^\dagger m_D)_{ii}}\sum_{j\neq i}\left[\mathcal{I}_{ij}^\alpha f\left(\frac{M_j^2}{M_i^2}\right) + \mathcal{J}_{ij}^\alpha\left(1 - \frac{M_j^2}{M_i^2}\right)^{-1}\right] \end{aligned} \quad (3.7)$$